

If the material is preliminarily loaded, then by virtue of the orientation of the structural elements (presumably sooty structures) strengthening of the material takes place. This in turn leads to an increase in the height of the potential barrier and a deviation from Eq. (4).

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FRACTURE OF MATERIAL THROUGH THE ACTION OF AN INTERNAL HEAT SOURCE

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UDC 536.421.1

An analytic study was made of the displacement of the vaporization front and of the resultant thermoelastic stresses in a semiinfinite solid through the action of an internal spherical heat source.

A heat source with a maximum at a certain depth is formed through the action of an electron beam, as noted in [1-3]. In principle, much of the same mechanism of energy deposition is possible for a pulsed laser focused at some depth in the material, for a pulsed discharge with the Lenz-Joule effect predominant, etc. With sufficiently sharp focusing and significant intensity of the heat source, volume vaporization is possible at artificial centers (fluctuation and gas bubbles, inclusions, etc.) which can lead to additional removal of mass as noted in [4].

We consider the following as a model problem for the analysis of the physical processes occurring through the action of such heat sources. Through the action of an enclosed volume heat source, vaporization of material takes place immediately and a high-temperature vapor region is created which occupies a sphere of radius R_0 at the initial time. Heat transfer from this region gives rise to heating of the solid with subsequent phase transition (vaporization). With this kind of physics for the process, volume vapor formation occurs in the medium at high pressure. The kinetics of such a process was not developed. Therefore, Frenkel' kinetics [5] was chosen at the zeroth approximation for this work. Displacement of the phase-transition boundary is determined from a solution of the Stefan problem (the effect of a free surface is not considered):

$$\frac{\partial T}{\partial t} = \frac{a}{r^2} \cdot \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right), \quad R_0 + \int_0^t v(\tau) d\tau \leq r < \infty, \quad (1)$$

$$M(t) c_v \frac{dT_v}{dt} = H(t) \left(T_{r=R_0 + \int_0^t v(\tau) d\tau} - T_v \right), \quad r < R_0 + \int_0^t v(\tau) d\tau, \quad (2)$$

$$-\lambda \left(\frac{dT}{dr} \right)_{r=R_0 + \int_0^t v(\tau) d\tau} = \frac{H(t)}{4\pi \left(R_0 + \int_0^t v(\tau) d\tau \right)^2} \left(T_v - T_{r=R_0 + \int_0^t v(\tau) d\tau} \right) - \rho L v(t), \quad (3)$$

Vitevsk Technological Institute of Light Industry. Translated from Inzhenerno-Fizicheski Zhurnal, Vol. 32, No. 4, pp. 723-728, April, 1977. Original article submitted March 18, 1976.

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TABLE 1. Dependence of Time t_1 for Expansion of Vapor Bubble and of the Relative Increase in Radius $(R_{fr} - R_0)/R_0$ on the Initial Vapor Temperature T_0 .

| Material | T_0 , deg | $t_1 \cdot 10^3$ sec | $\frac{R_{fr} - R_0}{R_0}$ |
|----------|-------------|----------------------|----------------------------|
| Fe | 3800 | 2,8 | $4,3 \cdot 10^{-4}$ |
| Fe | 4800 | 3,3 | $4,6 \cdot 10^{-3}$ |
| Fe | 5850 | 3,8 | $4,9 \cdot 10^{-2}$ |
| W | 7600 | 9,1 | $7,4 \cdot 10^{-4}$ |
| W | 9100 | 10,8 | $8,4 \cdot 10^{-3}$ |
| W | 11400 | 11,9 | $8,7 \cdot 10^{-2}$ |

$$\left(\frac{dT}{dr} \right)_{r \rightarrow \infty} = 0, \quad (4)$$

$$T(r, 0) = 0, \quad R_0 \leq r < \infty, \quad (5)$$

$$T_v(r, 0) = \frac{Q}{4/3\pi R_0^3 c_v \rho_v} = T_0, \quad 0 < r < R_0, \quad (6)$$

$$v(t) = c_s \exp\left(-\frac{E}{T(r = R_0 + \int_0^t v(\tau) d\tau, t)} \right), \quad (7)$$

where $R(t) = 1/H(t)$ is the thermal resistance.

We consider the asymptotic solution of the problem formulated for the case where the advance of the vaporization front is small in comparison with the radius of the vapor bubble produced, i.e.,

$$\int_0^t v(\tau) d\tau \ll R_0. \quad (8)$$

We assume that the thermophysical characteristics of the material and of the vapor produced are identical and independent of temperature and we take for the thermal resistance of the contact between vapor and solid [6]

$$R(t) = \frac{1}{4\pi\lambda R_0}. \quad (9)$$

Using Eqs. (8) and (9), we then obtain integral equations for the phase-transition front R_{fr} in the form

$$R_0 T(R_{fr}, t) + \sqrt{\frac{a}{\pi}} \int_0^t \frac{T(R_{fr}, \tau)}{\sqrt{t-\tau}} d\tau = \sqrt{\frac{a}{\pi}} \int_0^t \frac{T_{fr}(\tau) - T(R_{fr}, \tau)}{\sqrt{t-\tau}} d\tau - \sqrt{\frac{a}{\pi}} \int_0^t \frac{R_0 \rho L v(\tau)}{\lambda \sqrt{t-\tau}} d\tau, \quad (10)$$

$$T_v(t) = T_0 \exp\left(-\frac{3at}{R_0^2} \right) + \int_0^t \frac{3a}{R_0^2} T(R_{fr}, \tau) \exp\left[-\frac{3a}{R_0^2}(t-\tau) \right] d\tau. \quad (11)$$

For most materials the characteristic temperature is

$$E \gg T\left(r = R_0 + \int_0^t v(\tau) d\tau, t \right), \quad (12)$$

and, applying the method of steepest descent to the term with $v(\tau)$ in Eqs. (10), (11) under the conditions (7) and (12), and then considering the asymptotic estimates of the temperature of the material at the phase-transition front for long times [7], we obtain

$$T(R_{fr}, t) \sim \frac{T_0 R_0^3}{6\sqrt{\pi}(at)^{3/2}} - \frac{Lc_0\sqrt{t}}{c\sqrt{\pi a}}. \quad (13)$$

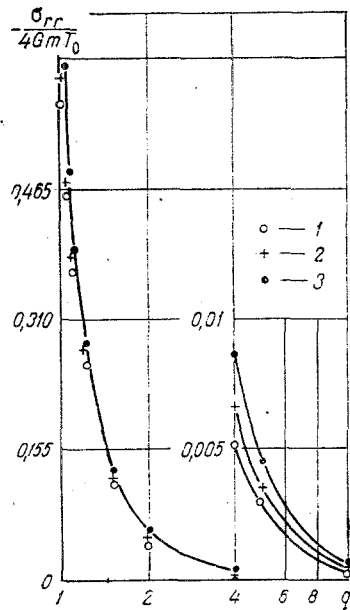


Fig. 1. Dependence of dimensionless stress $\sigma_{rr}/4GmT_0$ on the dimensionless distance $q = l/R_0$: 1) $\nu = 0.25$; 2) $\nu = 0.3$; 3) $\nu = 0.33$.

Physical considerations indicate that at some time the temperature at the phase-transition front $T(R_{fr}, t) \sim T_b$, i.e., all the heat will be used up in phase transition and thermal conductivity. The rate of displacement of the phase-transition boundary then tends to zero, and the maximum radius of the vapor bubble evaluated by the method of steepest descent under the conditions (7) and (12) is

$$R_{fr} = R_0 + \int_0^{t_1} v(\tau) d\tau \sim R_0 + c_0 t_1, \quad (14)$$

where

$$c_0 = c_s \exp\left(-\frac{E}{T_0}\right). \quad (15)$$

This time is determined from Eq. (13) where one should set

$$T(R_0 + c_0 t_1, t_1) = T_b, \quad (16)$$

where T_b is the boiling point corresponding to thermodynamic conditions at maximum expansion of the vapor bubble.

Asymptotic estimates of t_1 and of the advance of the vaporization front are given in Table 1 (Fe, W) for $R_0 \sim 10^{-7}$ m, and for T_b and the thermophysical constants of the materials under normal conditions. The expressions obtained for t_1 and for the maximum radius of the vapor bubble should be considered as upper limits, since the return flow of atoms because of condensation is not taken into account completely in this problem. In addition, under actual conditions, the nonequilibrium nature of the temperature field leads to bubble motion, which also changes its size.

If the vapor bubble produced is close to a surface, fracture of the material is possible through the combined action of vapor pressure and resultant thermoelastic stresses.

We evaluate the magnitude of the thermoelastic stresses at a surface of the material which is at a distance l from the center of a vapor bubble.

After maximum expansion of the vapor bubble, the temperature field in the material is described in the simplest case by the equation of thermal conductivity,

$$\frac{\partial T}{\partial t} = a \left[\frac{1}{r} \cdot \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right], \quad 0 \leq r, \quad z < \infty, \quad (17)$$

$$T(r, z, 0) = \begin{cases} T_b & \sqrt{r^2 + (z-l)^2} < R_0, \\ 0 & \sqrt{r^2 + (z-l)^2} \geq R_0, \end{cases} \quad (18)$$

TABLE 2. Dependence of the Dimensionless Time $x = lR_0/at$ on the Dimensionless Coordinate $q = lR_0$ for Different ν

| $q = \frac{l}{R_0}$ | $x = lR_0/at$ | | |
|---------------------|---------------|-------------|--------------|
| | $\nu = 0,25$ | $\nu = 0,3$ | $\nu = 0,33$ |
| 1,01 | 301,336 | 321,253 | 333,208 |
| 1,05 | 61,3480 | 65,2650 | 67,6210 |
| 1,1 | 31,3623 | 33,2788 | 34,4333 |
| 1,2 | 16,3898 | 17,3066 | 17,8609 |
| 1,5 | 7,43830 | 7,76410 | 7,96203 |
| 2,0 | 4,33498 | 4,48853 | 4,58140 |
| 4,0 | 1,83196 | 1,88682 | 1,91981 |
| 5,0 | 1,44092 | 1,48333 | 1,50880 |
| 10,0 | 0,704951 | 0,725239 | 0,737416 |
| 50,0 | 0,0200019 | 0,0240045 | 0,0264067 |

$$\left(\frac{\partial T}{\partial z}\right)_{z=0} = \left(\frac{\partial T}{\partial z}\right)_{z \rightarrow \infty} = 0. \quad (19)$$

The solution of Eqs. (17)-(19) for $r > R_0$ is

$$T(r, z, t) = \varphi(r_1, t) + \varphi(r_2, t), \quad (20)$$

where

$$\varphi(r_1, t) = \frac{T_b}{2} \left\{ \Phi\left(\frac{r_1 + R_0}{2\sqrt{at}}\right) - \Phi\left(\frac{r_1 - R_0}{2\sqrt{at}}\right) \right\} - \frac{T_b}{r_1} \sqrt{\frac{at}{\pi}} \left\{ \exp\left[-\frac{(r_1 - R_0)^2}{4at}\right] - \exp\left[-\frac{(r_1 + R_0)^2}{4at}\right] \right\}, \quad (21)$$

$$r_1 = \sqrt{r^2 + (z - l)^2}; \quad r_2 = \sqrt{r^2 + (z + l)^2}. \quad (22)$$

Through the action of the temperature field, a field of thermoelastic stresses arises and the thermoelastic potential of the latter in the quasistatic formulation is determined from the solution of the following problem [8]:

$$\frac{1}{r} \cdot \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) + \frac{\partial^2 \Phi}{\partial z^2} = mT(r, z, t), \quad (23)$$

$$(\sigma_{zz})_{z=0} = (\sigma_{rz})_{z=0} = 0. \quad (24)$$

Finally, the value of the thermoelastic stresses at the surface in the dimensionless variables

$$x = \frac{lR_0}{at}; \quad q = \frac{l}{R_0} \quad (25)$$

is given by

$$\begin{aligned} -\frac{\sigma_{rr}}{4GmT_b} &= \frac{1}{2q} \left[\operatorname{erfc} \frac{\sqrt{x}(q-1)}{2\sqrt{q}} + \operatorname{erfc} \frac{\sqrt{x}(q+1)}{2\sqrt{q}} \right] - \\ &- \frac{q+2\nu}{q\sqrt{xq}} i \operatorname{erfc} \frac{\sqrt{x}}{2\sqrt{q}}(q-1) + \frac{q-2\nu}{q\sqrt{xq}} i \operatorname{erfc} \frac{\sqrt{x}}{2\sqrt{q}}(q+1) + \\ &+ \frac{4\nu(q-1)}{xq^2} i^2 \operatorname{erfc} \frac{\sqrt{x}}{2\sqrt{q}}(q-1) - \frac{4\nu(q+1)}{xq^2} i^2 \operatorname{erfc} \frac{\sqrt{x}}{2\sqrt{q}}(q+1) + \\ &+ \frac{8\nu}{(qx)^{3/2}} \left[i^3 \operatorname{erfc} \frac{\sqrt{x}}{2\sqrt{q}}(q-1) - i^3 \operatorname{erfc} \frac{\sqrt{x}}{2\sqrt{q}}(q+1) \right] + \frac{2\nu}{3q^3}. \end{aligned} \quad (26)$$

In accordance with the quasistatic formulation of the problem, we have

$$-\frac{\sigma_{rr}}{4GmT_b}(t=0) = \frac{2\nu}{3q^3} \neq 0. \quad (27)$$

The asymptotes of Eq. (26) for short and long times are easily found from the asymptotic representation for $i^n \operatorname{erfc} z$ and are

$$-\frac{\sigma_{rr}}{4GmT_b} \sim \frac{2\nu}{3q^3} + \frac{1}{\sqrt{\pi xq}(q-1)} \exp\left[-\frac{x}{q}(q-1)^2\right] + \quad (28)$$

$t \rightarrow 0$

$$+ \frac{1}{\sqrt{\pi x q (q+1)}} \exp \left[-\frac{x}{q} (q+1)^2 \right], \quad (28)$$

$$\frac{\sigma_{rr}}{4GmT_b} \sim \frac{v \sqrt{x} (q^2+1)}{3 \sqrt{\pi} q^{5/2}}, \quad \sigma_{rr} \rightarrow 0, \quad (29)$$

$$t > \frac{lr_0}{a}.$$

Physical considerations indicate that the stress at the surface must be a maximum at some time. Taking the derivative of Eq. (26), we determine this time from the following transcendental equation (in the dimensionless variables):

$$(q-1)x^2 - 2x(q+2v) + 8v + [x^2(q+1) + 2x(q-2v) - 8v] \exp(-x) = 0. \quad (30)$$

The resultant equation has a single real root, whose value, calculated for different v and q on a Minsk-32 computer, are given in Table 2. Maximum values of the dimensionless stress at the surface calculated from Eq. (26) for different v and q are shown in Fig. 1 on a semi-logarithmic scale.

The equations obtained make it possible to estimate the size of the region which will be fractured under the action of thermoelastic stresses. It is necessary to know the tensile strength of the material for this. We assume that fracture of the material occurs at stresses of the order of kY . The theoretical value is $k \sim 0.1$, but in practice fracture occurs at values of k that are 3-5 times less. We then obtain from Eq. (26) and Fig. 1 the value of q for which σ_{rr} is equal to the tensile strength of the test material.

NOTATION

T , temperature; t , time; α , thermal diffusivity; λ , thermal conductivity; ρ , density; r, z , running coordinates; $M(t) = \frac{4}{3} \pi \left(R_0 + \int_0^t v(\tau) d\tau \right)^3 \rho_n$; c_v, T_v, ρ_v , mass specific heat, temperature, and density of vaporized material; $v(t)$, velocity of vaporization front; L , specific heat of vaporization; T_0 , initial temperature of vaporized material; Q , energy per pulse; $Y = E_1/R$, characteristic temperature in Frenkel' kinetics; T_b , boiling point; ϕ , thermoelastic potential; σ_{rz}, σ_{zz} , thermoelastic stresses; $m = [(1+\nu)/(1-\nu)]\alpha_t$, where ν is the Poisson coefficient and α_t is the coefficient of linear expansion; G , shear modulus; Y , Young's modulus.

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